



# Conformal Ricci Soliton on 3-dimensional trans-Sasakian manifolds with respect to SVK connection

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In this study, we adapt the Schouten-van Kampen (SVK) connection on trans-Sasakian 3-manifolds. Then we consider semi-symmetric conditions on trans-Sasakian 3-manifolds with respect to the SVK connection admitting conformal Ricci soliton.

*Key words and phrases:* Ricci soliton, conformal Ricci soliton, Schouten-van Kampen connection, trans-Sasakian 3-manifold.

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## 1 Introduction

In 1982, the concept of Ricci flow was introduced by R.S. Hamilton [12]. This concept was developed to answer Thurston's geometric conjecture, which says that each closed three manifold admits a geometric decomposition. Also, R.S. Hamilton classified all compact manifolds with positive curvature operator in dimension four [12]. The Ricci flow equation is given by

$$\frac{\partial g}{\partial t} = -2S,$$

on a compact Riemannian manifold  $M$  with Riemannian metric  $g$ .

A self-similar soliton to the Ricci flow [12, 28] is known a Ricci soliton [13], if it moves only by a one parameter family of diffeomorphism and scaling. The Ricci soliton equation is given by

$$\mathcal{L}_V g + 2S = 2\lambda g,$$

where  $\mathcal{L}_V$  is the Lie derivative,  $S$  is Ricci tensor,  $g$  is a Riemannian metric,  $V$  is a vector field and  $\lambda$  is a scalar. The Ricci soliton is said to be shrinking if  $\lambda$  is positive, steady if  $\lambda$  is zero and expanding if  $\lambda$  is negative. Many studies on Ricci solitons have been reported by many geometers in different structure (see [3, 4, 7, 8, 14, 29, 30]).

The concept of conformal Ricci flow was studied by A.E. Fischer [10]. The conformal Ricci flow on  $M$  is defined by the equation

$$\frac{\partial g}{\partial t} + 2 \left( S + \frac{g}{n} \right) = -pg$$

and  $r(g) = -1$ , where  $p$  is a scalar non-dynamical field (time dependent scalar field),  $r(g)$  is the scalar curvature of the manifold and  $n$  is the dimension of manifold [10].

Also, the notion of conformal Ricci soliton equation is given by

$$\mathcal{L}_V g + 2S = \left[ 2\lambda - \left( p + \frac{2}{n} \right) \right] g, \tag{1}$$

where  $\lambda$  is constant [1]. The equation is the generalization of the Ricci soliton equation and it also satisfies the conformal Ricci flow equation. In [21], on a 3-dimensional trans-Sasakian manifold conformal Ricci soliton was studied.

Moreover, the Schouten-van Kampen (SVK) connection defined as adapted to a linear connection for studying non holonomic manifolds and it is one of the most natural connections on a differentiable manifold [2, 15, 22]. A.F. Solov'ev studied hyperdistributions in Riemannian manifolds using the SVK connection [23, 25–27]. Then Z. Olszak studied SVK connection to almost (para) contact metric structures [17]. In recent times, S.Y. Perkaş and A. Yıldız studied some symmetry conditions and some soliton types of quasi-Sasakian manifolds and  $f$ -Kenmotsu manifolds with respect to SVK connection [19, 20].

In this study, we consider some curvature conditions on a 3-dimensional trans-Sasakian manifolds with respect to the SVK connection admitting conformal Ricci soliton and give an example of a 3-dimensional trans-Sasakian manifold with respect to SVK connection.

## 2 Preliminaries

Let  $M$  be an almost contact metric manifold with an almost contact metric structure  $(\phi, \xi, \eta, g)$ , where  $\phi$  is a (1,1) tensor field,  $\xi$  is a vector field,  $\eta$  is a 1-form and  $g$  is the compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \tag{2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{3}$$

$$g(X, \phi Y) = -g(\phi X, Y), \tag{4}$$

$$g(X, \xi) = \eta(X)$$

for all vector fields  $X, Y \in \chi(M)$ .

An almost contact metric structure  $(\phi, \xi, \eta, g)$  on  $M$  is called a trans-Sasakian structure [18] if  $(M \times R, J, G)$  belongs to the class  $W_4$  [11], where  $J$  is the almost complex structure on  $M \times R$  defined by  $J\left(X, f\frac{d}{dt}\right) = \left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right)$  for smooth functions  $f$  on  $M \times R$ .

It can be expressed [5] by the condition

$$(\nabla_X \phi) Y = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X) \tag{5}$$

for some smooth functions  $\alpha, \beta$  on  $M$  and we say that the trans-Sasakian structure is of type  $(\alpha, \beta)$ .

From the above expression, we have

$$\nabla_X \xi = -\alpha\phi X + \beta(X - \eta(X)\xi), \tag{6}$$

$$(\nabla_X \eta) Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y). \tag{7}$$

For a 3-dimensional trans-Sasakian manifold the following relations hold [9]:

$$2\alpha\beta + \xi\alpha = 0,$$

$$S(X, Y) = \left(\frac{r}{2} + \xi\beta - (\alpha^2 - \beta^2)\right)g(X, Y) - \left(\frac{r}{2} + \xi\beta - 3(\alpha^2 - \beta^2)\right)\eta(X)\eta(Y) - (Y\beta + (\phi Y)\alpha)\eta(X) - (X\beta + (\phi X)\alpha)\eta(Y),$$

$$S(X, \xi) = \left(2(\alpha^2 - \beta^2) - \xi\beta\right)\eta X - X\beta - (\phi X)\alpha,$$

where  $S$  denotes the Ricci tensor of type  $(0, 2)$ ,  $r$  is the scalar curvature of the manifold  $M$ .

For constant  $\alpha, \beta$ , the following relations hold [9]:

$$S(X, Y) = \left(\frac{r}{2} - (\alpha^2 - \beta^2)\right)g(X, Y) - \left(\frac{r}{2} - 3(\alpha^2 - \beta^2)\right)\eta(X)\eta(Y),$$

$$S(X, \xi) = 2(\alpha^2 - \beta^2)\eta(X),$$

$$R(X, Y)\xi = (\alpha^2 - \beta^2)[\eta(Y)X - \eta(X)Y],$$

$$R(\xi, X)Y = (\alpha^2 - \beta^2)[g(X, Y)\xi - \eta(Y)X], \tag{8}$$

$$\eta(R(X, Y)Z) = (\alpha^2 - \beta^2)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)].$$

From (5) it follows that if  $\alpha$  and  $\beta$  are constants, then the manifold is either  $\alpha$ -Sasakian or  $\beta$ -Kenmotsu or cosymplectic, respectively.

Also, we have two naturally defined distributions in the tangent bundle  $TM$  of  $M$  as follows

$$\tilde{H} = \ker \eta, \quad \tilde{V} = \text{span } \xi,$$

which implies  $TM = \tilde{H} \oplus \tilde{V}$ ,  $\tilde{H} \cap \tilde{V} = 0$  and  $\tilde{H} \perp \tilde{V}$ . This decomposition allows one to define the Schouten-van Kampen connection  $\tilde{\nabla}$  over an almost contact metric structure. The Schouten-van Kampen connection  $\tilde{\nabla}$  on an almost contact metric manifold with respect to Levi-Civita connection  $\nabla$  is defined [24] by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(Y)\nabla_X \xi + (\nabla_X \eta)(Y)\xi. \tag{9}$$

Moreover

$$\begin{aligned} (\mathcal{L}_\xi g)(X, Y) &= (\nabla_\xi g)(X, Y) - \alpha g(\phi X, Y) + 2\beta g(X, Y) - 2\beta \eta(X)\eta(Y) - \alpha g(\phi X, Y) \\ &= 2\beta g(X, Y) - 2\beta \eta(X)\eta(Y). \end{aligned}$$

Let  $M$  be a 3-dimensional trans-Sasakian manifold with constant  $\alpha$  and  $\beta$  with respect to SVK connection. Then using (6) and (7) in (9), we get

$$\tilde{\nabla}_X Y = \nabla_X Y + \alpha\{\eta(Y)\phi X - g(\phi X, Y)\xi\} + \beta\{g(X, Y)\xi - \eta(Y)X\}. \tag{10}$$

Let  $R$  and  $\tilde{R}$  be the curvature tensors of the Levi-Civita connection  $\nabla$  and SVK connection  $\tilde{\nabla}$  are given by

$$R(X, Y) = [\nabla_X, \nabla_Y] - \nabla_{[X, Y]}, \quad \tilde{R}(X, Y) = [\tilde{\nabla}_X, \tilde{\nabla}_Y] - \tilde{\nabla}_{[X, Y]}.$$

Using (10), we obtain

$$\begin{aligned} \tilde{R}(X, Y)Z = R(X, Y)Z + \alpha^2 \left\{ \begin{aligned} &g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y + \eta(X)\eta(Z)Y \\ &- \eta(Y)\eta(Z)X - g(Y, Z)\eta(X)\xi + g(X, Z)\eta(Y)\xi \end{aligned} \right\} \\ + \beta^2 \{g(Y, Z)X - g(X, Z)Y\}. \end{aligned} \tag{11}$$

We will also consider the Riemannian curvature tensors  $\tilde{R}, R$ , the Ricci tensors  $\tilde{S}, S$ , the Ricci operators  $\tilde{Q}, Q$  and the scalar curvatures  $\tilde{\tau}, \tau$  of the connections  $\tilde{\nabla}$  and  $\nabla$  are given by

$$\begin{aligned} \tilde{R}(X, Y, W, Z) = R(X, Y, W, Z) + \alpha^2 \left\{ \begin{aligned} &g(\phi Y, W)g(\phi X, Z) - g(\phi X, W)g(\phi Y, Z) \\ &+ g(Y, Z)\eta(X)\eta(W) - g(X, Z)\eta(Y)\eta(W) \\ &- g(Y, W)\eta(X)\eta(Z) + g(X, W)\eta(Y)\eta(Z) \end{aligned} \right\} \\ + \beta^2 \{g(Y, W)g(X, Z) - g(X, W)g(Y, Z)\}, \\ \tilde{S}(X, Y) = S(X, Y) + 2\beta^2 g(X, Y) - 2\alpha^2 \eta(X)\eta(Y), \\ \tilde{Q}X = QX + 2\beta^2 X - 2\alpha^2 \eta(X)\xi, \\ \tilde{\tau} = \tau - 2\alpha^2 + 6\beta^2. \end{aligned} \tag{12}$$

In a 3-dimensional trans-Sasakian manifold  $M$  endowed with respect to the SVK connection bearing a conformal Ricci soliton, we can write

$$\tilde{\mathcal{E}}_V g + 2\tilde{S} - \left\{ 2\lambda - \left( p + \frac{2}{3} \right) \right\} g = 0. \tag{13}$$

From (9) and (13), since  $\tilde{\nabla}g = 0$  and  $\tilde{T} \neq 0$ , we have

$$\tilde{\mathcal{E}}_V g(X, Y) = g(\nabla_X V, Y) + g(X, \nabla_Y V) = (\mathcal{L}_V g)(X, Y),$$

that is

$$g(\nabla_X V, Y) + g(X, \nabla_Y V) + 2\tilde{S}(X, Y) - \left\{ 2\lambda - \left( p + \frac{2}{3} \right) \right\} g(X, Y) = 0. \tag{14}$$

Putting  $V = \xi$  in (14), we obtain

$$g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi) + 2\tilde{S}(X, Y) - \left\{ 2\lambda - \left( p + \frac{2}{3} \right) \right\} g(X, Y) = 0. \tag{15}$$

Now using (6) with (4) in (15), we get

$$2\beta g(X, Y) - 2\beta \eta(X)\eta(Y) + 2\tilde{S}(X, Y) - \left\{ 2\lambda - \left( p + \frac{2}{3} \right) \right\} g(X, Y) = 0,$$

which yields

$$\tilde{S}(X, Y) = \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) - \beta \right\} g(X, Y) + \beta \eta(X)\eta(Y). \tag{16}$$

Thus  $M$  is an  $\eta$ -Einstein manifold with respect to SVK connection.

Also using (12) in (16), we have

$$S(X, Y) = \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) - \beta - 2\beta^2 \right\} g(X, Y) + (\beta + 2\beta^2) \eta(X)\eta(Y). \tag{17}$$

Hence  $M$  is an  $\eta$ -Einstein manifold with respect to the Levi-Civita connection. Thus we have the following assertion.

**Theorem 1.** *Let  $M$  be a 3-dimensional trans-Sasakian manifold admitting a conformal Ricci soliton  $(g, \xi, \lambda)$  with respect to SVK connection. Then  $M$  is an  $\eta$ -Einstein manifold both with respect to the SVK connection and Levi-Civita connection.*

Putting  $Y = \xi$  and using (12) in (16), we get

$$\tilde{S}(X, \xi) = \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right\} \eta(X). \quad (18)$$

So we discuss the following conditions:

- i) assume that  $p > \frac{2}{3}$  and therefore the equation (18) shows that  $\lambda > 0$ , thus the Ricci soliton  $(g, \xi, \lambda)$  is expanding;
- ii) assume that  $p < \frac{2}{3}$  and therefore the equation (18) reveals that  $\lambda < 0$ , thus the Ricci soliton  $(g, \xi, \lambda)$  is shrinking;
- iii) assume that  $p = \frac{2}{3}$  and therefore the equation (18) allows that  $\lambda = 0$ , thus the Ricci soliton  $(g, \xi, \lambda)$  is steady.

Thus we state our results in the form of theorem as follows.

**Theorem 2.** *A conformal Ricci soliton  $(g, \xi, \lambda)$  on a 3-dimensional trans-Sasakian manifold with respect to SVK connection is said to be expanding, shrinking and steady if  $p > \frac{2}{3}$ ,  $p < \frac{2}{3}$  and  $p = \frac{2}{3}$ , respectively.*

### 3 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold satisfying $R(\xi, X) \cdot \tilde{B} = 0$

The C-Bochner curvature tensor  $\tilde{B}$  in  $M$  is defined [16] (see also [6]) by

$$\begin{aligned} \tilde{B}(X, Y)Z = \tilde{R}(X, Y)Z + \frac{1}{6} & \left\{ \begin{array}{l} g(X, Z)\tilde{Q}Y - g(Y, Z)\tilde{Q}X \\ -\tilde{S}(Y, Z)X + \tilde{S}(X, Z)Y \\ +g(\phi X, Z)\tilde{Q}\phi Y - g(\phi Y, Z)\tilde{Q}\phi X \\ -\tilde{S}(\phi Y, Z)\phi X + \tilde{S}(\phi X, Z)\phi Y \\ +2\tilde{S}(\phi X, Y)\phi Z + 2g(\phi X, Y)\tilde{Q}\phi Z \\ +\eta(Y)\eta(Z)\tilde{Q}X - \eta(X)\eta(Z)\tilde{Q}Y \\ -\eta(Y)\tilde{S}(X, Z)\xi + \eta(X)\tilde{S}(Y, Z)\xi \end{array} \right\} \\ & - \frac{D+2}{6} \left\{ \begin{array}{l} g(\phi X, Z)\phi Y - g(\phi Y, Z)\phi X \\ +2g(\phi X, Y)\phi Z \end{array} \right\} \\ & + \frac{D}{6} \left\{ \begin{array}{l} \eta(Y)g(X, Z)\xi - \eta(Y)\eta(Z)X \\ +\eta(X)\eta(Z)Y - \eta(X)g(Y, Z)\xi \end{array} \right\} \\ & - \frac{D-4}{6} \left\{ g(X, Z)Y - g(Y, Z)X \right\}, \end{aligned}$$

where  $D = \frac{\tilde{r}+2}{6}$ .

Using (11) with (12) in the above equation, we get

$$\begin{aligned}
 \tilde{B}(X, Y)Z = & R(X, Y)Z + \alpha^2 \left\{ \begin{array}{l} g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ +\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ -g(Y, Z)\eta(X)\xi + g(X, Z)\eta(Y)\xi \end{array} \right\} \\
 & + \beta^2 \{g(Y, Z)X - g(X, Z)Y\} \\
 & + \frac{1}{6} \left\{ \begin{array}{l} S(X, Z)Y - S(Y, Z)X \\ +g(X, Z)QY - g(Y, Z)QX \\ +g(\phi X, Z)Q\phi Y - g(\phi Y, Z)Q\phi X \\ +S(\phi X, Z)\phi Y - S(\phi Y, Z)\phi X \\ -S(X, Z)\eta(Y)\xi + S(Y, Z)\eta(X)\xi \\ +2S(\phi X, Y)\phi Z + 2g(\phi X, Y)Q\phi Z \\ +\eta(Y)\eta(Z)QX - \eta(X)\eta(Z)QY \\ +4\beta^2 g(X, Z)Y - 4\beta^2 g(Y, Z)X \\ +4\beta^2 g(\phi X, Z)\phi Y - 4\beta^2 g(\phi Y, Z)\phi X \\ -2\beta^2 g(X, Z)\eta(Y)\xi + 2\beta^2 g(Y, Z)\eta(X)\xi \\ +8\beta^2 g(\phi X, Y)\phi Z \\ +2\beta^2 \eta(Y)\eta(Z)X - 2\beta^2 \eta(X)\eta(Z)Y \\ +2\alpha^2 g(Y, Z)\eta(X)\xi - 2\alpha^2 g(X, Z)\eta(Y)\xi \\ +2\alpha^2 \eta(Y)\eta(Z)X - 2\alpha^2 \eta(X)\eta(Z)Y \end{array} \right\} \tag{19} \\
 & - \frac{D+2}{6} \left\{ \begin{array}{l} g(\phi X, Z)\phi Y - g(\phi Y, Z)\phi X \\ +2g(\phi X, Y)\phi Z \end{array} \right\} \\
 & + \frac{D}{6} \left\{ \begin{array}{l} g(X, Z)\eta(Y)\xi - \eta(Y)\eta(Z)X \\ +\eta(X)\eta(Z)Y - g(Y, Z)\eta(X)\xi \end{array} \right\} \\
 & - \frac{D-4}{6} \{g(X, Z)Y - g(Y, Z)X\}.
 \end{aligned}$$

Taking  $Z = \xi$  in (19), we get

$$\tilde{B}(X, Y)\xi = \left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} [\eta(X)Y - \eta(Y)X], \tag{20}$$

which gives

$$\eta(\tilde{B}(X, Y)Z) = \left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} \left[ \begin{array}{l} g(X, Z)\eta(Y) \\ -g(Y, Z)\eta(X) \end{array} \right].$$

Now, we assume that the condition  $R(\xi, X).\tilde{B} = 0$  is satisfied, then we get

$$R(\xi, X)\tilde{B}(Y, Z)W - \tilde{B}(R(\xi, X)Y, Z)W - \tilde{B}(Y, R(\xi, X)Z)W - \tilde{B}(Y, Z)R(\xi, X)W = 0. \tag{21}$$

Using (8) in (21), we obtain

$$\begin{aligned}
 \eta(\tilde{B}(Y, Z)W, X) - g(\tilde{B}(Y, Z)W, X)\xi + g(X, Y)\tilde{B}(\xi, Z)W - \eta(Y)\tilde{B}(X, Z)W \\
 + g(X, Z)\tilde{B}(Y, \xi)W - \eta(Z)\tilde{B}(Y, X)W + g(X, W)\tilde{B}(Y, Z)\xi - \eta(W)\tilde{B}(Y, Z)X = 0.
 \end{aligned}$$

By taking inner product with  $\xi$  and using (2), we have

$$\begin{aligned} \eta(\tilde{B}(Y, Z)W)\eta(X) - g(\tilde{B}(Y, Z)W, X)\xi + g(X, Y)\eta(\tilde{B}(\xi, Z)W) - \eta(Y)\eta(\tilde{B}(X, Z)W) \\ + g(X, Z)\eta(\tilde{B}(Y, \xi)W) - \eta(Z)\eta(\tilde{B}(Y, X)W) \\ + g(X, W)\eta(\tilde{B}(Y, Z)\xi) - \eta(W)\eta(\tilde{B}(Y, Z)X) = 0. \end{aligned} \quad (22)$$

By using (20) in (22), we arrive at

$$\left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} \begin{bmatrix} g(Y, W)g(X, Z) \\ -g(Z, W)g(X, Y) \end{bmatrix} - g(\tilde{B}(Y, Z)W, X) = 0. \quad (23)$$

Now using (19) in (23), we get

$$\begin{aligned} & \left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} \begin{bmatrix} g(Y, W)g(X, Z) \\ -g(Z, W)g(X, Y) \end{bmatrix} - g(R(Y, Z)W, X) \\ & - \alpha^2 \left\{ \begin{array}{l} g(\phi Z, W)g(\phi Y, X) - g(\phi Y, W)g(\phi Z, X) \\ +g(X, Z)\eta(Y)\eta(W) - g(X, Y)\eta(Z)\eta(W) \\ -g(Z, W)\eta(Y)\eta(X) + g(Y, W)\eta(Z)\eta(X) \end{array} \right\} \\ & - \beta^2 \{ g(Z, W)g(X, Y) - g(Y, W)g(X, Z) \} \\ & - \frac{1}{6} \left\{ \begin{array}{l} S(X, Z)g(Y, W) - S(Z, W)g(Y, X) \\ +S(Y, W)g(X, Z) - S(Y, X)g(Z, W) \\ +S(\phi Z, X)g(\phi Y, W) - S(\phi Z, W)g(\phi Y, X) \\ -S(\phi Y, X)g(\phi Z, W) + S(\phi Y, W)g(\phi Z, X) \\ +2S(\phi Y, Z)g(\phi W, X) + 2S(\phi W, X)g(\phi Y, Z) \\ +S(Y, X)\eta(Z)\eta(W) - S(Y, W)\eta(Z)\eta(X) \\ +S(Z, W)\eta(Y)\eta(X) - S(Z, X)\eta(Y)\eta(W) \\ +4\beta^2 g(Y, W)g(X, Z) - 4\beta^2 g(Z, W)g(Y, X) \\ -2\alpha^2 g(Y, W)\eta(X)\eta(Z) + 2\alpha^2 g(X, Y)\eta(Z)\eta(W) \\ +2\alpha^2 g(Z, W)\eta(X)\eta(Y) - 2\alpha^2 g(X, Z)\eta(Y)\eta(W) \\ +4\beta^2 g(\phi Y, W)g(\phi Z, X) - 4\beta^2 g(\phi Z, W)g(\phi Y, X) \\ +4\beta^2 g(\phi Y, Z)g(\phi W, X) \\ +2\beta^2 g(Y, X)\eta(Z)\eta(W) - 2\beta^2 g(Y, W)\eta(Z)\eta(X) \\ +2\beta^2 g(Z, W)\eta(Y)\eta(X) - 2\beta^2 g(Z, X)\eta(Y)\eta(W) \end{array} \right\} \\ & + \frac{D+2}{6} \left\{ \begin{array}{l} g(\phi Y, W)g(\phi Z, X) - g(\phi Z, W)g(\phi Y, X) \\ +2g(\phi Y, Z)g(\phi W, X) \end{array} \right\} \\ & - \frac{D}{6} \left\{ \begin{array}{l} g(Y, W)\eta(Z)\eta(X) - \eta(Z)\eta(W)g(X, Y) \\ -g(W, Z)\eta(X)\eta(Y) + \eta(Y)\eta(Z)g(X, Z) \end{array} \right\} \\ & + \frac{D-4}{6} \{ g(Y, W)g(Z, X) - g(Z, W)g(Y, X) \} = 0. \end{aligned}$$

Taking  $Y = X = e_i$  and summing over  $i = 1, 2, 3$ , where  $e_i$  is an orthonormal basis of  $T_p M$ , we obtain

$$\begin{aligned} S(Z, W) &= \left\{ \begin{array}{l} \frac{\tau}{6} - \beta + \beta^2 - \alpha^2 \\ +\frac{2}{3} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) - \frac{2D}{3} - 1 \end{array} \right\} g(Z, W) \\ &+ \left\{ \begin{array}{l} \frac{2D}{3} + 1 - \frac{\tau}{6} \\ -\frac{2}{3} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + 5\alpha^2 - 3\beta^2 \end{array} \right\} \eta(Z)\eta(W). \end{aligned}$$

Taking  $Z = W = \zeta$  in the above equation with using (17), we arrive at

$$\lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) = 2 \left( \alpha^2 - \beta^2 \right).$$

Since  $\alpha^2 \neq \beta^2$ , in this case we have the following.

- i) Assume that  $\alpha^2 \geq \beta^2$ , then  $(\alpha - \beta)(\alpha + \beta) \geq 0$ , which implies  $\alpha$  greater than  $\beta$ . So,  $\lambda > 0$  and Ricci soliton is shrinking.
- ii) Assume that  $\alpha^2 < \beta^2$  and  $p + \frac{2}{3} > 4(\alpha^2 - \beta^2)$ , then  $(\alpha - \beta)(\alpha + \beta) < 0$ , which gives  $\alpha$  less than  $-\beta$ . So,  $\lambda > 0$  and Ricci soliton is shrinking.
- iii) Assume that  $\alpha^2 < \beta^2$  and  $p + \frac{2}{3} < 4(\alpha^2 - \beta^2)$ , then  $(\alpha - \beta)(\alpha + \beta) < 0$ , which gives  $\alpha$  less than  $-\beta$ . So,  $\lambda < 0$  and Ricci soliton is expanding.

**Theorem 3.** A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies  $R(\zeta, X).\tilde{B} = 0$  and admits conformal Ricci soliton, then

- i) for  $\alpha > \beta$ , the Ricci soliton is shrinking;
- ii) for  $\alpha < -\beta$  and  $p + \frac{2}{3} > 4(\alpha^2 - \beta^2)$ , the Ricci soliton becomes shrinking;
- iii) for  $\alpha < -\beta$  and  $p + \frac{2}{3} < 4(\alpha^2 - \beta^2)$ , the Ricci soliton becomes expanding.

#### 4 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{B}(\zeta, X).S = 0$

The condition  $\tilde{B}(\zeta, X).S = 0$  implies that

$$S(\tilde{B}(\zeta, X)Y, Z) + S(Y, \tilde{B}(\zeta, X)Z) = 0. \tag{24}$$

Using (17) in (24), we get

$$\left\{ \begin{array}{l} \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \\ -\beta - 2\beta^2 \end{array} \right\} \left[ \begin{array}{l} g(\tilde{B}(\zeta, X)Y, Z) \\ +g(Y, \tilde{B}(\zeta, X)Z) \end{array} \right] + \{2\alpha^2 + \beta\} \left[ \begin{array}{l} \eta(\tilde{B}(\zeta, X)Y)\eta(Z) \\ +\eta(\tilde{B}(\zeta, X)Z)\eta(Y) \end{array} \right] = 0.$$

By using (20) in the above equation, we obtain

$$\{2\alpha^2 + \beta\} \left\{ \begin{array}{l} \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \\ -\beta - 2\beta^2 \end{array} \right\} \left[ \begin{array}{l} 2\eta(X)\eta(Y)\eta(Z) \\ -g(X, Y)\eta(Z) - g(X, Z)\eta(Y) \end{array} \right] = 0.$$

Taking  $Y = X = e_i$  and summing over  $i = 1, 2, 3$ , where  $e_i$  is an orthonormal basis of  $T_pM$  and taking condition  $\eta(Z) \neq 0$ , we obtain

$$\{2\alpha^2 + \beta\} \left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} = 0.$$

Thus we get the following result.

**Theorem 4.** A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies  $\tilde{B}(\zeta, X).S = 0$  and admits conformal Ricci soliton with  $2\alpha^2 \neq \beta$ , then

- i) if  $p = \frac{22}{3}$ , then  $\lambda = 0$  and Ricci soliton is steady;
- ii) if  $p < \frac{22}{3}$ , then  $\lambda < 0$  and Ricci soliton is expanding;
- iii) if  $p > \frac{22}{3}$ , then  $\lambda > 0$  and Ricci soliton is shrinking.



## 5 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{S}(\xi, X).\tilde{R} = 0$

If we consider the condition  $\tilde{S}(\xi, X).\tilde{R} = 0$ , then we have

$$\begin{aligned} (\tilde{S}(\xi, X).\tilde{R})(U, V)W &= \tilde{S}(\xi, \tilde{R}(U, V)W)X - \tilde{S}(X, \tilde{R}(U, V)W)\xi + \tilde{S}(\xi, U)\tilde{R}(X, V)W \\ &\quad - \tilde{S}(\xi, U)\tilde{R}(\xi, V)W + \tilde{S}(\xi, V)\tilde{R}(U, X)W - \tilde{S}(\xi, V)\tilde{R}(U, \xi)W \\ &\quad + \tilde{S}(\xi, W)\tilde{R}(U, V)X - \tilde{S}(X, W)\tilde{R}(U, V)\xi. \end{aligned} \quad (25)$$

By using (16) in (25), we have

$$\begin{aligned} \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right\} &\left[ \begin{array}{l} \eta(\tilde{R}(U, V)W)X + \eta(U)\tilde{R}(X, V)W \\ + \eta(V)\tilde{R}(U, X)W + \eta(W)\tilde{R}(U, V)X \end{array} \right] \\ &- \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) - \beta \right\} \left[ \begin{array}{l} g(X, \tilde{R}(U, V)W)\xi + g(X, U)\tilde{R}(\xi, V)W \\ + g(X, V)\tilde{R}(U, \xi)W + g(X, W)\tilde{R}(U, V)\xi \end{array} \right] \\ &- \beta \left[ \begin{array}{l} \eta(X)\eta(\tilde{R}(U, V)W)\xi + \eta(X)\eta(U)\tilde{R}(\xi, V)W \\ + \eta(X)\eta(V)\tilde{R}(U, \xi)W + \eta(X)\eta(W)\tilde{R}(U, V)\xi \end{array} \right] = 0. \end{aligned}$$

By taking inner product with  $\xi$  and using (11), we get

$$\left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right\} \left[ \begin{array}{l} g(X, \tilde{R}(U, V)W) \\ + \alpha^2 \left( \begin{array}{l} g(\phi V, W)g(\phi U, X) - g(\phi U, W)g(\phi V, X) \\ + g(X, V)\eta(U)\eta(V) - g(X, U)\eta(X)\eta(W) \\ - g(V, W)\eta(U)\eta(X) + g(U, W)\eta(V)\eta(X) \end{array} \right) \\ + \beta^2 (g(V, W)g(X, U) - g(U, W)g(X, V)) \end{array} \right] = 0.$$

Taking  $X = U = e_i$  and summing over  $i = 1, 2, 3$ , where  $e_i$  is an orthonormal basis of  $T_pM$ , we find

$$\left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right\} \left[ \begin{array}{l} S(V, W) \\ + \alpha^2 (g(V, W) - 4\eta(V)\eta(W)) \\ + 2\beta^2 g(V, W) \end{array} \right] = 0.$$

Taking  $V = W = \xi$  in the above equation, we arrive at

$$\left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right\} \left\{ \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) - \alpha^2 \right\} = 0,$$

which gives either  $\lambda = \frac{1}{2} \left( p + \frac{2}{3} \right)$  or  $\lambda = \alpha^2 + \frac{1}{2} \left( p + \frac{2}{3} \right)$ .

So we can give the following result.

**Theorem 5.** A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies  $\tilde{S}(\xi, X).\tilde{R} = 0$  and admits conformal Ricci soliton, then

*Case I:*  $\lambda$  depends only on  $p$

- i) if  $p = -\frac{2}{3}$ , then  $\lambda = 0$  and Ricci soliton is steady;
- ii) if  $p > -\frac{2}{3}$ , then  $\lambda > 0$  and Ricci soliton is shrinking;
- iii) if  $p < -\frac{2}{3}$ , then  $\lambda < 0$  and Ricci soliton is expanding;

*Case II:*  $\lambda$  depends on both  $p$  and  $\alpha$

- i) if  $p = -\frac{2}{3} - 2\alpha^2$ , then  $\lambda = 0$  and Ricci soliton is steady;
- ii) if  $p > -\frac{2}{3} - 2\alpha^2$ , then  $\lambda > 0$  and Ricci soliton is shrinking;
- iii) if  $p < -\frac{2}{3} - 2\alpha^2$ , then  $\lambda < 0$  and Ricci soliton is expanding.

### 6 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{B}.\phi = 0$

Now, we examine the curvature condition  $\tilde{B}.\phi = 0$  on a 3-dimensional trans-Sasakian manifold with respect to SVK connection admitting a conformal Ricci soliton.

We know that if  $\tilde{B}.\phi = 0$ , then we have

$$(\tilde{B}.\phi)(X, Y)Z = 0,$$

which gives

$$\tilde{B}(X, Y)\phi Z - \phi\tilde{B}(X, Y)Z = 0. \tag{26}$$

If we take  $Z = \xi$  in (26), we find

$$\phi\tilde{B}(X, Y)\xi = 0. \tag{27}$$

Using (20) in (27), we obtain

$$\left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} [\eta(X)\phi Y - \eta(Y)\phi X] = 0. \tag{28}$$

Replacing  $X$  by  $\phi X$  in (28) and using (2), we get

$$\left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} [(-X + \eta(X))\eta(Y)] = 0. \tag{29}$$

Taking  $Y = \xi$  and replacing  $X$  by  $\phi X$  in (29), we have

$$\left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} \phi X = 0.$$

Finally, taking inner product with  $W$ , we obtain

$$\left\{ \frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} g(\phi X, W) = 0.$$

It follows that

$$\frac{1}{6} \left( \lambda - \frac{1}{2} \left( p + \frac{2}{3} \right) \right) + \frac{4}{6} = 0.$$

So we can state the following result.

**Theorem 6.** *A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies  $\tilde{B}.\phi = 0$  and admits conformal Ricci soliton, then*

- i) if  $p = \frac{22}{3}$ , then  $\lambda = 0$  and Ricci soliton is steady;
- ii) if  $p < \frac{22}{3}$ , then  $\lambda < 0$  and Ricci soliton is expanding;
- iii) if  $p > \frac{22}{3}$ , then  $\lambda > 0$  and Ricci soliton is shrinking.

**Example 1.** We consider a 3-dimensional manifold  $= \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 \neq 0\}$ , where  $(x_1, x_2, x_3)$  are standard coordinates in  $\mathbb{R}^3$ . Let  $\{\omega_1, \omega_2, \omega_3\}$  be a linearly independent global frame on  $M$  defined by

$$\omega_1 = e^{-\omega_3} \left( \frac{\partial}{\partial \omega_1} + \frac{\partial}{\partial \omega_2} \right), \quad \omega_2 = e^{-\omega_3} \left( -\frac{\partial}{\partial \omega_1} + \frac{\partial}{\partial \omega_2} \right), \quad \omega_3 = \frac{\partial}{\partial \omega_3}.$$

Let  $g$  be the Riemannian metric defined by

$$g(\omega_1, \omega_1) = g(\omega_2, \omega_2) = g(\omega_3, \omega_3) = 1, \\ g(\omega_1, \omega_2) = g(\omega_1, \omega_3) = g(\omega_2, \omega_3) = 0.$$

If  $\eta$  is the 1-form defined by  $\eta(W) = g(W, \omega_3)$  and if  $\phi$  is the  $(1, 1)$ -tensor field defined by

$$\phi(\omega_1) = \omega_2, \quad \phi(\omega_2) = -\omega_1, \quad \phi(\omega_3) = 0.$$

Also, we have

$$\eta(\omega_3) = 1, \\ \phi Y = -Y + \eta(Y)\xi, \\ g(\phi Y, \phi V) = g(Y, V) + \eta(Y)\eta(V).$$

Now, we get

$$[\omega_1, \omega_2] = 0, \quad [\omega_1, \omega_3] = 0, \quad [\omega_2, \omega_3] = 0.$$

From Koszul's formula, we obtain

$$\begin{aligned} \nabla_{\omega_1}\omega_1 &= -\omega_3, & \nabla_{\omega_2}\omega_1 &= 0, & \nabla_{\omega_3}\omega_1 &= 0, \\ \nabla_{\omega_1}\omega_2 &= 0, & \nabla_{\omega_2}\omega_2 &= -\omega_3, & \nabla_{\omega_3}\omega_2 &= 0, \\ \nabla_{\omega_1}\omega_3 &= \omega_1, & \nabla_{\omega_2}\omega_3 &= \omega_2, & \nabla_{\omega_3}\omega_3 &= 0. \end{aligned} \quad (30)$$

In view of the above equations,  $(\phi, \xi, \eta, g)$  satisfy (2) and (3) with  $\alpha = 0$  and  $\beta = 1$ . So,  $M$  is a trans-Sasakian manifold [31]. In view of (30), we find

$$\begin{aligned} R(\omega_1, \omega_2)\omega_3 &= 0, & R(\omega_2, \omega_3)\omega_3 &= -\omega_2, & R(\omega_1, \omega_3)\omega_3 &= -\omega_1, \\ R(\omega_1, \omega_2)\omega_2 &= -\omega_1, & R(\omega_2, \omega_3)\omega_2 &= \omega_3, & R(\omega_1, \omega_3)\omega_2 &= 0, \\ R(\omega_1, \omega_2)\omega_1 &= \omega_2, & R(\omega_2, \omega_3)\omega_1 &= 0, & R(\omega_1, \omega_3)\omega_1 &= \omega_3. \end{aligned}$$

If we consider SVK connection to this equation from (10) with (30), we obtain

$$\begin{aligned} \tilde{\nabla}_{\omega_1}\omega_1 &= (\beta - 1)\omega_3, & \tilde{\nabla}_{\omega_2}\omega_1 &= \alpha\omega_3, & \tilde{\nabla}_{\omega_3}\omega_1 &= 0, \\ \tilde{\nabla}_{\omega_1}\omega_2 &= -\alpha\omega_3, & \tilde{\nabla}_{\omega_2}\omega_2 &= (\beta - 1)\omega_3, & \tilde{\nabla}_{\omega_3}\omega_2 &= 0, \\ \tilde{\nabla}_{\omega_1}\omega_3 &= (1 - \beta)\omega_1 + \alpha\omega_2, & \tilde{\nabla}_{\omega_2}\omega_3 &= (1 - \beta)\omega_2 + \alpha\omega_1, & \tilde{\nabla}_{\omega_3}\omega_3 &= 0. \end{aligned} \quad (31)$$

We know that  $M$  is a trans-Sasakian manifold with respect to SVK connection. In view of (31), we get

$$\begin{aligned} \tilde{R}(\omega_1, \omega_2)\omega_3 &= 0, & \tilde{R}(\omega_2, \omega_3)\omega_3 &= (\beta^2 - \alpha^2 - 1)\omega_2, \\ \tilde{R}(\omega_1, \omega_2)\omega_2 &= (\beta^2 - \alpha^2 - 1)\omega_1, & \tilde{R}(\omega_2, \omega_3)\omega_2 &= (1 + \alpha^2 - \beta^2)\omega_3, \\ \tilde{R}(\omega_1, \omega_2)\omega_1 &= (1 + \alpha^2 - \beta^2)\omega_2, & \tilde{R}(\omega_2, \omega_3)\omega_1 &= 0, \\ \tilde{R}(\omega_1, \omega_3)\omega_1 &= \omega_3, \\ \tilde{R}(\omega_1, \omega_3)\omega_2 &= 0, \\ \tilde{R}(\omega_1, \omega_3)\omega_3 &= (\beta^2 - \alpha^2 - 1)\omega_1. \end{aligned}$$

From the above equation, we arrive at

$$\tilde{S}(\omega_1, \omega_1) = 2(\beta^2 - 1), \quad \tilde{S}(\omega_2, \omega_2) = 2(\beta^2 - 1), \quad \tilde{S}(\omega_3, \omega_3) = 2(\beta^2 - \alpha^2 - 1).$$

Finally, in view of (1), if  $\lambda = 2\beta^2 - \beta + 2 + \frac{\beta}{2} + \frac{1}{3}$  and  $\beta = -2\alpha^2$ , then  $M$  admits a conformal Ricci soliton with respect to SVK connection.

## References

- [1] Basu N., Bhattacharyya A. *Conformal Ricci soliton in Kenmotsu manifold*. Glob. J. Adv. Res. Class. Mod. Geom. 2015, **4** (1), 15–21.
- [2] Bejancu A., Farran H. *Foliations and geometric structures*. In: Mathematics and Its Applications, 580. Springer, Dordrecht, 2006.
- [3] Blaga A.M., Yüksel P.S., Acet B.E., Erdoğan F.E.  *$\eta$ -Ricci solitons in  $\epsilon$ -almost paracontact metric manifolds*. Glas. Mat. 2018, **53** (1), 205–220. doi:10.3336/gm.53.1.14
- [4] Blaga A.M., Yüksel P.S. *Remarks on almost  $\eta$ -Ricci solitons in  $\epsilon$ -para Sasakian manifolds*. Comm. Fac. Sci. Uni. Ankara Series A1 Math. and Stat. 2019, **68** (2), 1621–1628. doi:10.31801/cfsuasmas.546595
- [5] Blair D.E., Oubina J.A. *Conformal and related changes of metric on the product of two almost contact metric manifolds*. Publ. Mat. 1990, **34** (1), 199–207.
- [6] Bochner S. *Curvature and Betti numbers II*. Ann. of Math. (2) 1949, **50** (2), 77–93. doi:10.2307/1969353
- [7] Chakraborty D., Mishra V.N., Hui S.K. *Ricci soliton on three-dimensional  $\beta$ -Kenmotsu manifolds with respect to Schouten-van Kampen connection*. J. Ultra Scientist of Phys. Sci. 2018, **30** (1), 86–91. doi:10.22147/jusps-A/300110
- [8] Chaubey S.K., De U.C. *Three-dimensional trans-Sasakian manifolds and solitons*. Arab. J. Math. Sci. 2022, **28** (1), 112–125. doi:10.1108/AJMS-12-2020-0127
- [9] Dutta T., Basu N., Bhattacharyya A. *Almost conformal Ricci solitons on 3-dimensional trans-Sasakian manifold*. Hacet. J. Math. Stat. 2016, **45** (5), 1379–1392. doi:10.15672/HJMS.20164514287
- [10] Fischer A.E. *An introduction to conformal Ricci flow*. Classical Quantum Gravity 2004, **21** (3), 171–218. doi:10.1088/0264-9381/21/3/011
- [11] Gray A., Hervella M.L. *The sixteen classes of almost Hermitian manifolds and their linear invariants*. Ann. Mat. Pura Appl. (4) 1980, **123** (4), 35–58. doi:10.1007/BF01796539
- [12] Hamilton R.S. *Three manifold with positive Ricci curvature*. J. Differential Geom. 1982, **17** (2), 255–306. doi:10.4310/jdg/1214436922
- [13] Hamilton R.S. *The Ricci flow on surfaces*. Contemp. Math. 1988, **71**, 237–261. doi:10.1090/conm/071/954419
- [14] Hui S.K., Chakraborty D. *Ricci almost soliton on concircular Ricci pseudosymmetric  $\beta$ -Kenmotsu manifolds*. Hacet. J. Math. Stat. 2018, **47** (3), 579–587. doi:10.15672/HJMS.2017.471
- [15] Ianus S. *Some almost product structures on manifolds with linear connection*. Kodai Math. Sem. Rep. 1971, **23**, 305–310.
- [16] Kim J.S., Tripathi M.M., Choi J.-D. *On C-Bochner curvature tensor of a contact metric manifold*. Bull. Korean Math. Soc. 2005, **42** (4), 713–724. doi:10.4134/bkms/2005.42.4.713
- [17] Olszak Z. *The Schouten-van Kampen affine connection adapted an almost (para) contact metric structure*. Publ. de l'Inst. Math. 2013, **94** (108), 31–42. doi:10.2298/PIM1308031O
- [18] Oubina J.A. *New classes of almost contact metric structures*. Publ. Math. Debrecen 1985, **32** (4), 187–193.
- [19] Perktas S.Y., Yıldız A. *On quasi-Sasakian 3-manifolds with respect to the Schouten-van Kampen connection*. Int. Electron. J. Geom 2020, **13** (2), 62–74. doi:10.36890/iejg.742073
- [20] Perktas S.Y., Yıldız A. *On  $f$ -Kenmotsu 3-manifolds with respect to the Schouten-van Kampen connection*. Turkish J. Math. 2021, **45** (1), 387–409. doi:10.3906/mat-2003-121
- [21] Roy S., Bhattacharyya A. *Conformal Ricci soliton on 3-dimensional trans-Sasakian manifold*. Jordan J. Math. Stat. 2020, **13** (1), 89–109.
- [22] Schouten J., Van Kampen E. *Zur Einbettungs- und Krümmungstheorie nichtholonomer Gebilde*. Math. Ann. 1930, **103**, 752–783.
- [23] Solov'ev A.F. *On the curvature of the connection induced on a hyperdistribution in a Riemannian space*. Geom. Sb. 1982, **19**, 12–23. (in Russian)

- [24] Solov'ev A.F. *On the curvature of the connection induced on a hyperdistribution in a Riemannian space*. Geom. Sb. 1978, **19**, 12–23. (in Russian)
- [25] Solov'ev A.F. *The bending of hyperdistributions*. Geom. Sb. 1979, **20**, 101–112. (in Russian)
- [26] Solov'ev A.F. *Second fundamental form of a distribution*. Mat. Zametki 1982, **35**, 139–146.
- [27] Solov'ev A.F. *Curvature of a distribution*. Mat. Zametki 1984, **35**, 111–124.
- [28] Topping P. *Lecture on the Ricci Flow*. In: Süli E. (Eds.) London Mathematical Society Lecture Note Series, 325. Cambridge University Press, United Kingdom, 2006.
- [29] Turan M., De U.C., Yıldız A. *Ricci solitons and gradient Ricci solitons in 3-dimensional trans-Sasakian manifolds*. Filomat 2012, **26** (2), 363–370. doi:10.2298/FIL1202363T
- [30] Turan M., De U.C., Yıldız A., De A. *Ricci solitons and gradient Ricci solitons on 3-dimensional normal almost contact metric manifolds*. Publ. Math. Debrecen 2012, **80** (1-2), 127–142. doi:10.5486/PMD.2012.4947
- [31] Vishnuvardhana S.V., Venkatesha V., Turgut V.A. *On 3-dimensional trans-Sasakian manifold admitting a semi-symmetric metric connection*. GU J Sci. 2019, **32** (1), 242–254.

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У цьому дослідженні ми адаптуємо зв'язність Схаутен-ван Кампена (SVK) до тривимірних транс-Сасакаєвих многовидів. Також, ми розглядаємо напівсиметричні умови на тривимірних транс-Сасакаєвих многовидах відносно SVK зв'язності, що допускає конформний солітон Річчі.

*Ключові слова і фрази:* солітон Річчі, конформний солітон Річчі, зв'язність Схаутен-ван Кампена, тривимірний транс-Сасакаєвий многовид.